



# 部分多様体の幾何解析

## — Extrinsic Geometric Analysis —

### Abstracts

**June 9 (Mon.)**

Shin-ichi Ohta (Osaka University)

**Title:** Splitting theorems for weighted Finsler spacetimes

**Abstract:** A Finsler spacetime is a generalization of a usual (Lorentzian) spacetime in the same way that a Finsler manifold generalizes a Riemannian manifold, and a weighted Finsler spacetime is a Finsler spacetime equipped with a measure on it. In this general framework, inspired by a recently developed technique to deform a hyperbolic d’Alembertian into an elliptic operator by Braun et al, we establish timelike splitting theorems under nonnegative weighted Ricci curvature in timelike directions. This is joint work with Erasmo Caponio and Argam Ohanyan.

Tatsuya Miura (Kyoto University)

**Title:** A new energy method for shortening infinite-length curves

**Abstract:** The curve shortening flow is a prototypical geometric flow that can be interpreted as the gradient flow of the length functional. While the length-decreasing property is often a powerful tool in analyzing long-time behavior, it becomes ineffective in the case of curves with infinite length. In this talk, I will present recent joint work with Fabian Rupp (University of Vienna), in which we introduce a new energy method that reinterprets “curve shortening” as “tangent aligning”. This provides a robust variational approach to infinite-length curve shortening flows, circumventing the use of maximum principles, and yields a new convergence result to a straight line for initial data that may be non-planar and possess unbounded graphical ends.

Xiaoli Han (Tsinghua University)

**Title:** The deformed Hermitian Yang-Mills equation

**Abstract:** First I will introduce the definition of the deformed Hermitian Yang-Mills equation (It is also called LYZ equation). Then I will talk about the existence results of this equation. I will also talk about the parabolic equation associated with the deformed Hermitian Yang-Mills equation.

## June 10 (Tue.)

Yuanlin Peng (Tohoku University)

**Title:** Applications of entire Green functions on  $\mathrm{RCD}(0, N)$  spaces

**Abstract:** An  $\mathrm{RCD}(K, N)$  space is a metric measure space with Ricci curvature lower bounded by  $K$  and dimension upper bounded by  $N$ . This talk includes two applications of Green functions in the non-negative curvature case. The first one is a sharp gradient estimate and corresponding rigidity results, based on a joint work with Prof. Shouhei Honda, which generalized Colding's results in the manifold case. The second part is application in PDEs. That is, if the space admits an entire Green function with certain growth bound, then the Poisson equation  $\Delta u = f$  admits a continuous solution whenever  $f \in L^1_{loc}$  satisfies a decay bound in the integral sense. This is a generalization of Ni-Shi-Tam's results in the manifold case.

Shota Hamanaka (Osaka University)

**Title:** Applications of  $\mu$ -bubbles to studying GRS

**Abstract:** I will discuss two applications of  $\mu$ -bubbles to studying gradient Ricci solitons (GRS for short). One is to estimating the scalar curvature decay of nonparabolic steady GRS. Another is to estimating the diameter upper bound of a Riemannian manifold whose  $\infty$ -Bakry-Emery Ricci tensor is bounded below by a positive constant. (Although this diameter bound is not the best, it will be presented as an application of  $\mu$ -bubbles.)

Chao Xia (Xiamen University)

**Title:** Various Heintze-Karcher-type inequalities and applications

**Abstract:** Heintze-Karcher's inequality is a sharp geometric inequality in geometry of submanifolds. I will review the Heintze-Karcher's normal map method and Ros's method based on Reilly's formula. Then we give several generalizations in the case of hypersurfaces in manifolds with different curvature conditions, and in the case of hypersurfaces with capillary boundary.

Chao Li (New York University)

**Title:** The stable Bernstein theorem for minimal hypersurfaces

**Abstract:** The classical Bernstein problem, resolved in the 1960s, states that the only entire solution in  $\mathbb{R}^n$  to the minimal surface equation is affine whenever  $n$  is at most 7. A natural extension of the problem — the stable Bernstein problem for minimal hypersurfaces — has been extensively studied since 1980s. I will discuss recent progress on this problem: a complete, two-sided, codimension one stable minimal immersion in  $\mathbb{R}^n$ ,  $4 \leq n \leq 6$ , is flat. The solution relies on an improved understanding between curvature conditions and the macroscopic geometry of Riemannian manifolds.

Giovanni Catino (Politecnico di Milano)

**Title:** Stable minimal hypersurfaces via conformal method and applications

**Abstract:** In this talk I will describe some recent results concerning the rigidity of complete, immersed, stable (or delta-stable) minimal hypersurfaces of dimension three in the Euclidean space  $\mathbb{R}^4$ . The results rely on a conformal method, inspired by classical works of Schoen-Yau and Fischer-Colbrie. I will also present several applications of these techniques.

## June 11 (Wed.)

Shintaro Akamine (Nihon University)

**Title:** Bernstein type theorem for constant mean curvature surfaces in isotropic 3-space

**Abstract:** In this talk, we will discuss surfaces represented as graphs of some functions in the isotropic 3-space, which is the 3-space with a degenerate metric. In particular, we give a result on the value distribution of Gaussian curvature of complete constant mean curvature surfaces and show a Bernstein type theorem as an application of the result. This talk is based on the joint work with Wonjoo Lee and Seong-Deog Yang in Korea University.

Shin Nayatani (Nagoya University)

**Title:** First-eigenvalue maximization and inflation of maps

**Abstract:** Maximization problem for the first eigenvalue of the Laplacian began with the seminal work of Hersch (1970), who proved that on the two-sphere, the first eigenvalue (multiplied by area for scale invariance) is maximized by the round metrics (and by them only). Since then, this subject has been enriched by many interesting results. Meanwhile, in graph theory, Fiedler (1989) considered a similar maximization problem, and more recently, Göring-Helmberg-Wappler (2008, 2011) formulated a problem concerning realizations of a graph in Euclidean spaces, to which Fiedler's problem is 'dual'. In this talk, I will introduce an analogue of the GHW formalism in differential geometry. In fact, as a dual problem to an optimization problem concerning maps from a manifold into the Hilbert space  $l^2$ , we arrive at a first-eigenvalue maximization problem concerning the Bakry-Émery Laplacian rather than the usual Laplacian. I will discuss examples (tori, Berger spheres ...) and present an analogue of Nadirashvili's theorem, which asserts that a metric maximizing the first eigenvalue of the usual Laplacian admits an isometric minimal immersion into a sphere.

Reiko Miyaoka (Tohoku University)

**Title:** Chern's conjecture in the Dupin case

**Abstract:** Chern's conjecture states that a closed minimal hypersurface  $M^n$  in the sphere  $S^{n+1}$  is an isoparametric hypersurface if it has constant scalar curvature. This is still open when the number  $g$  of distinct principal curvatures is greater than 3, without further assumptions on the principal curvatures. In the case of Dupin hypersurfaces, which have constant principal curvatures with constant multiplicities along the curvature distribution, we obtain: A closed Dupin hypersurface with constant mean curvature is isoparametric when (1)  $g = 3$ , (2)  $g = 4$  and has constant scalar curvature or constant Lie curvature, (3)  $g = 6$  and has constant Lie curvatures. They cover all the non-trivial cases, where the Lie curvatures are given by the cross ratio among four distinct principal curvatures ([arXiv:2504.02621](#)).